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ON A NEW FORMULA FOR THE n-TH PRIME NUMBER

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There are some formulae for the n-th prime number as well as for function $\pi(n)$, determining the number of the prime numbers smaller than n (see, e.g. [1]). In [2] we introduced three new formulae for $\pi(n)$ and a new formula for the n-th prime number p_n. Now we shall introduce another – simpler formala for $\pi(n)$ and p_n, following [2].

Let us define functions sg and $\overline{sg}$ by:

$$sg(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1, & \text{if } x > 0 \end{cases}, \quad \overline{sg}(x) = \begin{cases} 0, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases},$$

where x is a real number.

For the natural number $n = \prod_{i=1}^{k} p_i^{\alpha_i}$, where $k, \alpha_1, \alpha_2, \ldots \alpha_k \geq 1$ are natural numbers and $p_1, p_2, \ldots, p_k$ are different prime numbers, let us define (see [3]) function η by:

$$\eta(n) = \sum_{i=1}^{n} \alpha_i . p_i .$$

THEOREM 1: The following equality holds for every natural number $n \geq 2$:

$$\pi(n) = \sum_{k=2}^{n} \overline{sg}(k - \eta(k)).$$

Proof: For every natural number k, such that $k \leq n$, if k is prime, then $\overline{sg}(k - \eta(k)) = 1$. On the other hand, if k is not prime, then $k - \eta(k) > 0$, i.e., $\overline{sg}(k - \eta(k)) = 0$. Therefore, the sum is equal to $\pi(n)$.

Of course, $\pi(0) = 0$ and $\pi(1) = 0$.

For the so constructed formula for $\pi(n)$, by analogy with [2] we can prove

THEOREM 2: For every natural number n:

$$p_n = \sum_{i=0}^{2^n} sg(n - \pi(i)).$$

REFERENCES:

[1] Ribenboim, P. *The New Book of Prime Number Records,* Springer, New York, 1995.

[2] Atanassov, K. A new formula for the n-th prime number. *Comptes Rendus de l'Academie Bulgare des Sciences,* Vol. 54, 2001, No. 7, 5-6.

[3] Atanassov K., Some assertions on "φ" and "σ" functions, *Bulletin of Number Theory and Related Topics,* Vol. XI (1987), No. 1, 50-63.