NNTDM 9 (2003) 2, 39-40

ON SOME PROBLEMS RELATED TO SMARANDACHE NOTIONS

Edited by M. Perez

1. Problem of Number Theory by L. Seagull, Glendale Community College

Let n be a composite integer > 4. Prove that in between n and $S(n)$ there exists at least a prime number.

Solution:

T.Yau proved that the Smarandache Function has the foliowing property: $S(n) \leq \frac{n}{2}$ for any composite number n, because: if $n = pq$, with $p < q$ and $(p, q) = 1$, then

$$S(n) \max(S(p), S(q)) = S(q) \leq q = \frac{n}{p} \leq \frac{n}{2}.$$

Now, using Bertrand-Tchebichev's theorem, we get that in between $\frac{n}{2}$ and n there exists at least a prime number.

2. Proposed Problem by Antony Begay

Let $S(n)$ be the smallest integer number such that $S(n)!$ is divisible by n, where $m! = 1.2.3.\ldots.m$ (factoriel of m), and $S(1) = 1$ (Smarandache Function). Prove that if p is prime then $S(p) = p$. Calculate $S(42)$.

Solotion:

$S(p)$ cannot be less than p, because if $S(p) = n < p$ then $n! = 1.2.3.\ldots.n$ is not divisible by p (p being prime). Thus $S(p) \geq p$. But $p! = 1.2.3.\ldots.p$ is divisible by p, and is the smallest one with this property. Therefore $S(p) = p$.

$42 = 2.3.7, 7! = 1.2.3.4.5.6.7$ which is divisible by 2. by 3, and by 7. Thus $S(42) \leq 7$. But $S(42)$ can not be less than 7, because for example $6! = 1.2.3.4.5.6$ is not divisible by 7. Hence $S(42) = 7$.

3. Proposed Problem by Leonardo Motta

Let n be a square free integer, and p the largest prime which devides n. Show that $S(n) = p$, where $S(n)$ is the Smarandache Function, i.e. the smallest integer such that $S(n)!$ is divisible by n.

Solution:

Because n is a square free number, there is no prime q such that q^2 divides n. Thus n is a product of distinct prime numbers, each one to the first power only. For example 105 is square free because 105=3.5.7 , i.e. 105 is a product of distinct prime numbers, each of them to the power 1 only. While 945 is not a square free number because $945 = 3^3.5.7$, therefore 945 is divisible by 3^2 (which is 9, i.e. a square). Now, if we compute the Smarandache Function $S(105) = 7$ because 7!=1.2.3.4.5.6.7 which is divisible by 3,5, and 7 in the same time, and 7 is smallest number with this property. But $S(945) = 9$, not 7. Therefore, if $n = a.b.\ldots.p$, where all $a < b < \ldots < p$ are distinct two by two primes, then $S(n) = \max(a, b, \ldots, p = p$, because the factorial of p, the largest prime which divides n, includes the factors $a, b, \ldots$ in its development: $p! = 1.\ldots.a.\ldots.b.\ldots.p$.

4. Proposed Problem by Gilbert Johnson

Let $Sdf(n)$ be the Smarandache Double Factorial Function, i.e. the smallest integer such that $Sdf(n)!!$ is divisible by n, where $m!! = 1.3.5.\ldots.m$ if m is odd and $m!! = 2.4.6.\ldots.m$ if m is even, If n is an even square free number and p the largest prime which divides n, then $Sdf(n) = 2p$.

Solution:

Because n is even and square free, then $n = 2.a.b.\ldots.p$ where all $2 < a < b < \ldots < p$ are distinct primes two by two, occuring to the power 1 only. $Sdf(n)$ cannot be less that $2p$ because if it is $2p - k$, with $1 \leq k < 2p$, then $(2p - k)!!$ would not be divisible by p.

$$(2p)!! = 2.4.\ldots.(2a).\ldots.(2b).\ldots.(2p)$$

is divisible by n and it is the smallest number with this property.