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AN ELEMENTARY EXTENSION OF HERMITE'S EQUALITY

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Ch. Hermite had introduced and proved the following well-known equality for each positive real number x and each natural number n:

$$[x] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + ... + [x + \frac{n-1}{n}] = [nx]. \tag{1}$$

We shall extend it, proving that for each positive real number x and each natural numbers k and n:

$$[x] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + ... + [x + \frac{kn-1}{n}] = k[nx] + \frac{(k-1)k}{n}. \tag{2}$$

In the proof of (2) we shall use (1). It is seen directly that

$$[x] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + ... + [x + \frac{kn-1}{n}]$$

$$= [x] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + ... + [x + \frac{n-1}{n}] + [x + \frac{n}{n}] + [x + \frac{n+1}{n}] + ... + [x + \frac{2n-1}{n}]$$

$$+ ... + [x + \frac{(k-1)n}{n}] + [x + \frac{(k-1)n+1}{n}] + ... + [x + \frac{kn-1}{n}]$$

$$= k([x] + [x + \frac{1}{n}] + [x + \frac{2}{n}] + ... + [x + \frac{n-1}{n}]) + n(0 + 1 + ... + (k-1))$$

$$= k[nx] + \frac{(k-1)k}{n}.$$

When $k = 1$ from (2) we obtain (1).