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SOME EXPLICIT FORMULAE FOR THE COMPOSITE NUMBERS

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Abstract: Explicit formulae for n-th term of the sequence of all composite numbers and for the sequence of all odd composite numbers are proposed.

§0.

In [1] three different formulae for n-th term C_n of an arbitrary increasing sequence $C = \{c_i\}_{i=1}^{\infty}$ of natural numbers are proposed. They are based on the numbers $\pi_C(k)$ ($k = 0, 1, 2, ...$), where $\pi_C(0) = 0$ and for $k \geq 1$ $\pi_C(k)$ denotes the number of terms of C, which are not greater than k. These formulae are givem below:

$$C_n = \sum_{k=0}^{\infty} [\frac{1}{1 + [\frac{\pi_C(k)}{n}]}]. \tag{1}$$

$$C_n = -2. \sum_{k=0}^{\infty} \zeta(-2.[\frac{\pi_C(k)}{n}]). \tag{2}$$

$$C_n = \sum_{k=0}^{\infty} \frac{1}{\Gamma(1 - [\frac{\pi_C(k)}{n}])}. \tag{3}$$

In (2) ζ means Riemann's function zeta. In (3) Γ means Euler's function gamma. Also $[x]$ denotes the largest integer, which is not greater than the real nonnegative number x.

If the inequality

$$C_n \leq \lambda_n$$

holds for every $n \geq 1$, where the numbers λ_n ($n = 1, 2, 3, ...$) are a priori known, than formulae (1) - (3) take the forms, respectively:

$$C_n = \sum_{k=0}^{\lambda_n} [\frac{1}{1 + [\frac{\pi_C(k)}{n}]}]. \tag{1'}$$

$$C_n = -2. \sum_{k=0}^{\lambda_n} \zeta(-2.[\frac{\pi_C(k)}{n}]). \tag{2'}$$

$$C_n = \sum_{k=0}^{\lambda_n} \frac{1}{\Gamma(1 - [\frac{\pi_C(k)}{n}])}. \tag{3'}$$

Using $(1')$ - $(3')$ the author have found in [1] three different explicit representations for n-th prime number p_n with $\lambda_n = n^2$, or with

$$\lambda_n = [\frac{n^2+3n+4}{4}]$$

(by choice).

Also, using (1) - (3), the author (again in [1]) have found three different explicit representations for $p_2(n)$, where $p_2(n)$ means n-th term of the sequence of twin primes. For example:

$$p_2(1) = 3,\ p_2(2) = 5,\ p_2(3) = 7,\ p_2(4) = 11,\ p_2(5) = 13,\ p_2(6) = 17,\ p_2(7) = 19, ...$$

§**1.**

Let C be the sequence of all composite numbers including 1 (because 1 is not included in the sequence of the prime numbers), i.e.:

$$c_1 = 1,\ c_2 = 4,\ c_3 = 6,\ c_4 = 8,\ c_5 = 9,\ c_6 = 10,\ c_7 = 12,\ c_8 = 14,\ c_9 = 15,\ c_{10} = 16, ...$$

It is trivial to see that for $k \geq 0$:

$$\pi_C(k) = k - \pi(k),$$

where $\pi(k)$ as usually means the number of the prime numbers that are not greater than k. Also, for $n \geq 1$ we have obviously:

$$C_n \leq \lambda_n$$

with $\lambda_n = 2n$.

Therefore, applying formulae $(1')$ - $(3')$ we obtain:

$$C_n = \sum_{k=0}^{2n} [\frac{1}{1+[\frac{\pi_C(k)}{n}]}]. \tag{5}$$

$$C_n = -2.\sum_{k=0}^{2n} \zeta(-2.[\frac{\pi_C(k)}{n}]). \tag{6}$$

$$C_n = \sum_{k=0}^{2n} \frac{1}{\Gamma(1-[\frac{\pi_C(k)}{n}])}. \tag{7}$$

§**2.**

Let C be the sequence of all odd composite numbers including 1, i.e.:

$$c_1 = 1,\ c_2 = 9,\ c_3 = 15,\ c_4 = 21,\ c_5 = 25, ...$$

It is clear that

$$\pi_C(0) = 0,\ \pi_C(1) = 1 \tag{8}$$

and for $k \geq 2$:

$$\pi_C(k) = k + 1 - [\frac{k}{2}] - \pi(k). \tag{9}$$

Also, for $n \geq 1$ the inequality

$$C_n \leq \lambda_n$$

holds for

$$\lambda_n = 3(2n - 1) = 6n - 3. \tag{10}$$

Therefore, applying formulae (1′) - (3′) and using (8) - (10), we obtain for $n \geq 2$:

$$C_n = 2 + \sum_{k=2}^{6n-3} [\frac{1}{1 + [\frac{k+1-[\frac{k}{2}]-\pi(k)}{n}]}],$$

$$C_n = 2 - 2.\sum_{k=2}^{6n-3} \zeta(-2.[\frac{k+1-[\frac{k}{2}]-\pi(k)}{n}]),$$

$$C_n = 2 + \sum_{k=2}^{6n-3} \frac{1}{\Gamma(1 - [\frac{k+1-[\frac{k}{2}]-\pi(k)}{n}])}.$$

It is possible to put $[\frac{k+3}{2}]$ instead of $k + 1 - [\frac{k}{2}]$ in the abive formulae.

Reference

[1] Vassilev-Missana, M. Three formulae for n-th prime and six for n-th term of twin primes, Notes on Number Theory and Discrete Mathematics, Vol. 7, 2001, No. 1, 15-20.