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ON THE 16-th SMARANDACHE'S PROBLEM

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In [1] Florian Smarandache formulated 105 unsolved problems.

The 16-th problem from [2] (see also 21-st problem from [1]) is the following:

Digital sum:

$$\underbrace{0,1,2,3,4,5,6,7,8,9}, \underbrace{1,2,3,4,5,6,7,8,9,10}, \underbrace{2,3,4,5,6,7,8,9,10,11},$$

$$\underbrace{3,4,5,6,7,8,9,10,11,12}, \underbrace{4,5,6,7,8,9,10,11,12,13}, \underbrace{5,6,7,8,9,10,11,12,13,14}, \ldots \qquad (1)$$

($d_s(n)$ is the sum of digits.)
Study this sequence.

First we shall note that function d_s is the first step of another arithmetic (digital) function φ, discussed in details in the author's paper [3].

After application of this function over the set of the natural numbers, or over the above sequence, we obtain the sequence

$$\underbrace{0,1,2,3,4,5,6,7,8,9}, \underbrace{1,2,3,4,5,6,7,8,9}, \underbrace{10,2,3,4,5,6,7,8,9}, \ldots$$

$$\underbrace{10,11,3,4,5,6,7,8,9}, \ldots \underbrace{10,11,12,4,5,6,7,8,9}, \ldots$$

On the other hand, in [3] another function (ψ) is introduced. After its application over the set of the natural numbers, or over the above sequence, we obtain the sequence

$$\underbrace{0,1,2,3,4,5,6,7,8,9}, \underbrace{1,2,3,4,5,6,7,8,9}, \underbrace{1,2,3,4,5,6,7,8,9}, \ldots$$

and the set $[1,2,3,4,5,6,7,8,9]$ is called a *basis* of the set of the natural numbers about ψ.

Below we shall show the form of the general term of the sequence from the Smarandache's problem. Let its members are denoted as $a_1, a_2, \ldots, a_n, \ldots$. The form of the member a_n is:

$$a_n = n - 9. \sum_{k=1}^{\infty} [\frac{n}{10^k}]. \qquad (2)$$

The validity of (2) can be proved, e.g., by induction. It is obviously valid for $n = 1$. Let us assume that for some n (2) is true. For n there are two cases.

Case 1: $n \neq \underbrace{9\,9\,...\,9}_{m \text{ times}}$ $(m \geq 1)$. Therefore

$$n + 1 \leq \underbrace{9\,9...\,9}_{m \text{ times}}$$

and

$$\sum_{k=1}^{\infty} [\frac{n}{10^k}] = \sum_{k=1}^{\infty} [\frac{n+1}{10^k}],$$

from where

$$a_{n+1} = a_n + 1 = n - 9. \sum_{k=1}^{\infty} [\frac{n}{10^k}] + 1 = (n+1) - 9. \sum_{k=1}^{\infty} [\frac{n+1}{10^k}].$$

Case 2: $n = \underbrace{9\,9\,...\,9}_{m \text{ times}}$. Therefore

$$n + 1 = 1\,\underbrace{0\,0...\,0}_{m \text{ times}}$$

and

$$a_{n+1} = 1 = 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - \underbrace{9\,9\,...\,9}_{m \text{ times}} = 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - 9.(1\,\underbrace{0\,0\,...\,0}_{m-1 \text{ times}} + 1\,\underbrace{0\,0\,...\,0}_{m-2 \text{ times}} + ...1)$$

$$= 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - 9. \sum_{k=1}^{\infty} [\frac{100...0}{10^k}] = (n+1) - 9. \sum_{k=1}^{\infty} [\frac{n+1}{10^k}].$$

Therefore (2) is true.

The second important question, which must be disccused about the sequence (1) is the validity of the equality $d_s(m) + d_s(n) = d_s(m+n)$. Obviously, it is not always valid. For example

$$d_s(2) + d_s(3) = 2 + 3 = 5 = d_s(5),$$

but

$$d_s(52) + d_s(53) = 7 + 8 = 15 \neq 6 = d_s(105).$$

The following assertion is true

$$d_s(m+n) = \begin{cases} d_s(m) + d_s(n), \text{ if } d_s(m) + d_s(n) \leq 9.\max([\frac{d_s(m)}{9}], [\frac{d_s(n)}{9}]) \\ d_s(m) + d_s(n) - 9.\max([\frac{d_s(m)}{9}], [\frac{d_s(n)}{9}]), \text{ otherwise} \end{cases}$$

The proof can be made again by the induction.

Let

$$R_k = k + (k+1) + \ldots + (k+9) = 10k + 45.$$

Obviously, R_k is the sum of the elements of the $k-$th group of (1).

Therefore, the sum of the first n members of (1) will be

$$S_n = \sum_{k=0}^{[\frac{n}{10}]-1} R_k + [\frac{n}{10}] + ([\frac{n}{10}] + 1) + \ldots + ([\frac{n}{10}] + n - 10.[\frac{n}{10}] - 1)$$

$$= 5.[\frac{n}{10}].([\frac{n}{10}] + 8) + (n - 10.[\frac{n}{10}]).[\frac{n}{10}] + \frac{1}{2}.(n - 10.[\frac{n}{10}]).(n - 10.[\frac{n}{10}] - 1),$$

i.e.,

$$S_n = 5.[\frac{n}{10}].([\frac{n}{10}] + 8) + (n - 10.[\frac{n}{10}]).(\frac{n-1}{2} - 4.[\frac{n}{10}]).$$

This equality can be proved directly or by induction.

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[1] F. Smarandache, Only problems, not solutions!. Xiquan Publ. House, Chicago, 1993.

[2] C. Dumitrescu, V. Seleacu, Some Sotions and Questions in Number Theory, Erhus Univ. Press, Glendale, 1994.

[3] K. Atanassov, An arithmetic function and some of its applications. Bull. of Number Theory and Related Topics Vol. IX (1985), No. 1, 18-27.