NNTDM 2 (1996) 4, 56-57

REMARK ON SET F_A

Krassimir T. Atanassov

Math. Research Lab., P.O.Box 12, Sofia-1113, BULGARIA

Let for a fixed natural number A, the set (see [1]):

$$F_A = \{x : \varphi(x) = A\}$$

be constructed, where for $A = \prod_{i=1}^{k} p_i^{\alpha_i}$, ($k$, α_1, α_2, ..., $\alpha_k \geq 1$ are natural numbers and p_1, p_2, ..., p_k are different prime numbers):

$\varphi(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1}.(p_i - 1)$ (see e.g. [2]).

Let for the above n: $set(n) = \{p_1, p_2, ..., p_k\}$.

We can note that the unique odd number for which $F_A \neq \emptyset$ is $A = 1$ and $F_1 = \{1, 2\}$.

Obviously, for every two even numbers A and B: $F_A \cap F_B = \emptyset$.

THEOREM 1 [1]: $\bigcup_{A \in E} F_A \cup \{1, 2\} = N$, where E is the set of the even numbers and N is the set of the natural numbers.

THEOREM 2: If $x \in F_A$ and $y \in F_B$ for some even numbers A and B then

$$x.y \in F_{A.B.C}, \text{ where } C = \prod_{p \in set(x) \cap set(y)} \frac{p}{p-1}.$$

Proof: Let $x \in F_A$ and $y \in F_B$, i.e., $\varphi(x) = A$ and $\varphi(y) = B$. Let

$x = \prod_{i=1}^{k+l} p_i^{\alpha_i}$ and $y = \prod_{i=1}^{k} p_i^{\beta_i} . \prod_{i=k+l+1}^{k+l+m} p_i^{\gamma_i}$. Therefore:

$$A = \prod_{i=1}^{k+l} p_i^{\alpha_i - 1}.(p_i - 1)$$

and

$$B = \prod_{i=1}^{k} p_i^{\beta_i - 1}.(p_i - 1). \prod_{i=k+l+1}^{k+l+m} p_i^{\beta_i - 1}.(p_i - 1).$$

Therefore

$$A.B = \prod_{i=1}^{k} p_i^{\alpha_i+\beta_i-2}.(p_i - 1). \prod_{i=k+1}^{k+l} p_i^{\alpha_i-1}.(p_i - 1).$$

$$\prod_{i=k+l+1}^{k+l+m} p_i^{\beta_i-1}.(p_i - 1)$$

and

$$\varphi(x.y) = \varphi(\prod_{i=1}^{k} p_i^{\alpha_i+\beta_i} . \prod_{i=k+1}^{k+l} p_i^{\alpha_i} . \prod_{i=k+l+1}^{k+l+m} p_i^{\beta_i})$$

$$= \prod_{i=1}^{k} p_i^{\alpha_i+\beta_i-1}.(p_i - 1). \prod_{i=k+1}^{k+l} p_i^{\alpha_i-1}.(p_i - 1). \prod_{i=k+l+1}^{k+l+m} p_i^{\beta_i-1}.(p_i - 1)$$

$$= A.B. \prod_{i=1}^{k} \frac{p_i}{p_i - 1} = A.B.C,$$

because $set(x) \cap set(y) = \{p_1, p_2, \ldots, p_k\}$.

The following assertions is proved analogically.

THEOREM 3: If $x \in F_A$ for some even number A, then for every natural number n: $x^n \in F_{A.x^{n-1}}$.

Different other properties of set F_A are discussed in [1].

REFERENCES:

[1] Atanassov K., Mihov S., Shannon A., Vassilev M., Some solved and unsolved problems on Euler's φ-function, submitted to Journal of Number Theory.

[2] Nagell T., Introduction to number theory, John Wiley & Sons, New York, 1950.

Received in BNT in June 1993