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COMPUTER CALCULUS RELATED TO SOME PROBLEMS IN NUMBER THEORY. 3

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Function ψ is defined in [1] by $\psi(n) = \varphi_s(n)$, where s is this smallest natural number for which $\varphi_s(n) = \varphi_{s+1}(n)$ for given natural number n and $\varphi_0(n) = n$ and $\varphi_{s+1}(n) = \varphi(\varphi_s(n))$, where $\varphi(0) = 0$ and $\varphi(m) \equiv \varphi(a_1 a_2 \ldots a_q) = \sum_{i=1} a_i$.

Some of its applications are shown in [1-4]. A relation between the Fibonacci numbers and ψ-function is shown in [1]. There, the following definition is introduced, too.

Let the sequence $a_1, a_2, \ldots$ (a_i are natural numbers for every i) be given and let $c_i = \psi(a_i)$ for every natural number i. If k and l exist such that $l \geq 0$, $c_{i+l} = c_{k+i+l} = c_{2.k+i+l} = \ldots$ for $1 \leq i \leq k$, then we shall say that $[c_{l+1}, c_{l+2}, \ldots, c_{l+k}]$ is a base of the sequence $\{c_i\}_i$ (the brief sign for sequence $c_1, c_2, \ldots$) with a length k and with respect to function ψ.

Here we shall discuss some new such relations.

Let everywhere k and n be natural numbers and let n be a fixed one. Let function G be defined by $G(n, 0) = \psi(f_{\psi(n)})$ and $G(n, k+1) = \psi(f_{\psi(G(n, k))})$. The first values of $G(n, k)$ are given in Table 1.

TABLE 1

n	ψ(n)	G(n, 0)	G(n, 1)	G(n, 2)	G(n, 3)
0	0	0	0	0	0
1	1	1	1	1	1
2	2	1	1	1	1
3	3	2	1	1	1
4	4	3	2	1	1
5	5	5	5	5	5
6	6	8	3	3	1
7	7	4	3	1	1
8	8	3	2	1	1
9	9	7	4	2	1
10	1	1	1	1	1
11	2	1	1	1	1
12	3	2	1	1	1
13	4	3	2	1	1
14	5	5	5	5	5
15	6	8	3	3	1
16	7	4	3	1	1
17	8	3	2	1	1
18	9	7	4	2	1
19	1	1	1	1	1
20	2	1	1	1	1
21	3	2	1	1	1
22	4	3	2	1	1
23	5	5	5	5	5
24	6	8	3	3	1
25	7	4	3	1	1
26	8	3	2	1	1
27	9	7	4	2	1
28	1	1	1	1	1
29	2	1	1	1	1
30	3	2	1	1	1

The validity of the following assertion follows from Table 1 and e.g. by induction.

<u>THEOREM 1</u>: (a) $\{G(n, 0)\}_n$ has a base $[1, 1, 2, 3, 5, 8, 4, 3, 7]$ with a length 9;

(b) $\{G(n, 1)\}_n$ has a base $[1, 1, 1, 2, 5, 3, 3, 2, 4]$ with a length 9;

(c) $\{G(n, 2)\}_n$ has a base $[1, 1, 1, 1, 5, 1, 1, 1, 2]$ with a length 9;

(d) $\{G(n, k)\}_n$ has a base $[1, 1, 1, 1, 5, 1, 1, 1, 1]$ with a length 9, for every natural number $k \geq 3$.

Another problem related to the above one is the following.

Let function E be defined by $E(n, 0) = \psi(f_n)$ and $E(n, k+1) = \psi(f_{\psi(E(n, k))})$. The first values of $E(n, k)$ are given in Table 2.

TABLE 2

n	E(n, 0)	E(n, 1)	E(n, 2)	E(n, 3)	E(n, 4)	E(n, 5)
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	2	1	1	1	1	1
4	3	2	1	1	1	1
5	5	5	5	5	5	5
6	8	3	2	1	1	1
7	4	3	2	1	1	1
8	3	2	1	1	1	1
9	7	4	3	2	1	1
10	1	1	1	1	1	1
11	8	3	2	1	1	1
12	9	7	4	3	2	1
13	8	3	2	1	1	1
14	8	3	2	1	1	1
15	7	4	3	2	1	1
16	6	8	3	2	1	1
17	4	3	2	1	1	1
18	1	1	1	1	1	1
19	5	5	5	5	5	5
20	6	8	3	2	1	1
21	2	1	1	1	1	1
22	8	3	2	1	1	1
23	1	1	1	1	1	1
24	9	7	4	3	2	1
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	2	1	1	1	1	1
28	3	2	1	1	1	1
29	5	5	5	5	5	5
30	8	3	2	1	1	1
40	4	3	2	1	1	1
41	3	2	1	1	1	1
42	7	4	3	2	1	1
43	1	1	1	1	1	1
44	8	3	2	1	1	1
45	9	7	4	3	2	1
46	8	3	2	1	1	1
47	8	3	2	1	1	1
48	7	4	3	2	1	1
49	6	8	3	2	1	1
50	4	3	2	1	1	1
51	1	1	1	1	1	1
52	5	5	5	5	5	5
53	6	8	3	2	1	1

54	2	1	1	1	1	1
55	8	3	2	1	1	1
56	1	1	1	1	1	1
57	9	7	4	3	2	1
58	1	1	1	1	1	1
59	1	1	1	1	1	1
60	2	1	1	1	1	1

The validity of the following assertion follows from Table 2 and e.g. by induction.

THEOREM 2: (a) $\{E(n, 0)\}_n$ has a base [1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9] with a length 24;

(b) $\{E(n, 1)\}_n$ has a base [1, 1, 1, 2, 5, 3, 3, 2, 4, 1, 3, 7, 3, 3, 4, 8, 3, 1, 5, 8, 1, 3, 1, 7] with a length 24;

(c) $\{E(n, 2)\}_n$ has a base [1, 1, 1, 1, 5, 2, 2, 1, 3, 1, 2, 4, 2, 2, 3, 3, 2, 1, 5, 3, 1, 2, 1, 4] with a length 24;

(d) $\{E(n, 3)\}_n$ has a base [1, 1, 1, 1, 5, 1, 1, 1, 2, 1, 1, 3, 1, 1, 2, 2, 1, 1, 5, 2, 1, 1, 1, 3] with a length 24;

(e) $\{E(n, 4)\}_n$ has a base [1, 1, 1, 1, 5, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 5, 1, 1, 1, 1, 2] with a length 24;

(f) $\{E(n, k)\}_n$ has a base [1, 1, 1, 1, 5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 5, 1, 1, 1, 1, 1] with a length 24, for every natural number $k \geq 5$.

The following PROBLEM is very interesting: to determine the base of sequence $\{\psi(D(n, k))\}_n$ for the different values of k, where $D(n, 0) = \psi(f_n)$ and $D(n, k+1) = \psi(f_{D(n, k)})$ if it there exists.

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