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# ON AN ARITHMETIC FUNCTION

Lyuben Karagyozov

11 Sofroniy Str., Bourgas-8000, BULGARIA

It is well known that for each two positive integers a, b, if $(a, b) = 1$, the equality

$$a.\mu - b.\gamma = 1 \tag{*}$$

has a solution for the integers $\mu$ and $\gamma$ (see, e.g., [1]).

Let $(\mu_0, \gamma_0) \in Z^2$ (where Z is the set of the integers) be a solution of (*). For every $c, \alpha \in N$ (where N is the set of the natural numbers), the equality

$$a.\mu - b.\gamma = \alpha.c$$

has integer solutions, too. For example: $\mu = \mu_0.\alpha.c$, $\gamma = \gamma_0.\alpha.c$.

<u>Definition 1</u>: If $a, b, c, d \in N$ and $(a, b) = (a.b, c) = 1$, then by $l_{a,b,c}(\alpha)$ we shall denote:

(a) the remain of division of $\mu = \mu_0.\alpha.c$ into b, if b is not a divisor of $\mu$; and

(b) b, if b is a divisor of $\mu$, where $(\mu_0, \gamma_0)$ is an arbitrary solution of (*).

Let everywhere below, a, b and c satisfy the above conditions. Therefore, $l_{a,b,c}(\alpha)$ is a positive integer and $l_{a,b,c}(\alpha) \in [1, b]$.

<u>THEOREM 1</u>: The value of $l_{a,b,c}(\alpha)$ is independent on the choise of the solution $(\mu_0, \gamma_0)$ of (*).

Proof: Let $(\mu_1, \gamma_1)$ and $(\mu_2, \gamma_2)$ be two different solutions of (*). Let $l_i$ is the value of $l_{a,b,c}(\alpha)$ for $(\mu_i, \gamma_i)$ for $i = 1, 2$. Therefore, there exist integer numbers $k_1$ and $k_2$ such that $l_i = \mu_i.\alpha.c-$

$K_i.b$. Let $\gamma^i = \gamma_i.\alpha.c - K_i.a$. Obviously $l_i$ and $\gamma^i$ are integers and

$$a.l_i - b.\gamma^i = a.(\mu_i.\alpha.c - K_i.b) - b.(\gamma_i.\alpha.c - K_i.a)$$

$$= \alpha.c.(a.\mu_i - b.\gamma_i) = \alpha c,$$

from where $a.(l_1 - l_2) = b.(\gamma^1 - \gamma^2)$ and from $(a, b) = 1$ it follows that $b$ is a divisor of $l_1 - l_2$. From the fact that $l_1, l_2 \in [1, b]$ it follows directly, that $l_1 = l_2$, with which the theorem is proved.

Analogically, the following assertions are proved.

<u>THEOREM 2</u>: Function $l_{a,b,c}$ is a bijection on $Y = [1, b]$ over $Y$.

<u>THEOREM 3</u>: For every natural number $\alpha$, $l_{a,b,c}(\alpha + b) = l_{a,b,c}(\alpha)$

<u>THEOREM 4</u>: For every two positive integers $\alpha_1$ and $\alpha_2$:

$$l_{a,b,c}(\alpha_1 + \alpha_2) = \begin{cases} l_{a,b,c}(\alpha_1) + l_{a,b,c}(\alpha_2), & \text{if } l_{a,b,c}(\alpha_1) + l_{a,b,c}(\alpha_2) \in Y \\ l_{a,b,c}(\alpha_1) + l_{a,b,c}(\alpha_2) - b, & \text{otherwise} \end{cases}$$

<u>THEOREM 5</u>: For every two positive integers $\alpha$ and $K$:

$$l_{a,b,c}(\alpha.K) = \begin{cases} K.l_{a,b,c}(\alpha), & \text{if } K.l_{a,b,c}(\alpha) \in Y \\ K.l_{a,b,c}(\alpha) - b.s, & \text{otherwise} \end{cases},$$

where $s = [K.l_{a,b,c}(\alpha)/b]$

finally, we shall note that the following question is interesting: Can the arithmetic function $\psi$ defined in [2] be represented as a particular case of function $l_{a,b,c}$.

REFERENCES:

[1] Nagell T., Introduction to Number Theory, Almqvist & Wiksell, Stockholm; John Willey & Sons, Inc., New York, 1950.

[2] Atanassov K., An arithmetic function and some of its applications, Bulletin of Number Theory and Related Topics, Vol. IX (1985), No. 1, 18-27.

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